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A Fuzzy Logic Based Approach to the SLAM Problem Using Pseudolinear Models with Multiframe Data Association

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Abstract

This paper presents an alternative solution to simultaneous localization and mapping (SLAM) problem by applying a fuzzy Kalman filter using pseudolinear process and measurement models. Takagi-Sugeno (T-S) fuzzy model based on observation for nonlinear system is adopted to represent the process and measurement models of the vehicle-landmarks system. Using the Kalman filter theory, each local T-S model is filtered to find the local estimates. The linear combination of these local estimates gives the global estimate for the complete system. The simulation results prove that the new approach results in more anticipated performances, though nonlinearity is directly involved in the Kalman filter equations, compared to the conventional approach.

1 Introduction

The simultaneous localization and mapping (SLAM) [1] problem, also known as concurrent mapping and localization (CML) problem, is often recognized in the robotics literature as one of the key challenges in building autonomous capabilities for mobile vehicles. The goal of an autonomous vehicle performing SLAM is to start from an unknown location in an unknown environment and build a map (consisting of environmental features) of its environment incrementally by using the uncertain information extracted from it sensors, whilst simultaneously using that map to localize itself with respect to a reference coordinate frame and navigate in real time.

The first solution to the SLAM problem was proposed by Smith *et al.* [2]. They emphasized the importance of map and vehicle correlations in SLAM and introduced the extended Kalman filter (EKF)-based stochastic mapping framework, which estimated the vehicle pose and the map feature (landmark) positions in an augmented state vector using second order statistics. Although the EKF-based SLAM within the stochastic mapping framework gained wide popularity among the SLAM research community. Over time, it was shown to have several shortcomings. Notable shortcomings are its susceptibility to data-association errors and inconsistent treatment of nonlinearities.

Here we propose some remedies to overcome the shortcomings of the EKF algorithm. To preserve the nonlinearity in the system, motion and observation models are represented by the pseudolinear models. Discrete time motion model is derived from the dead-reckoned measurements of the vehicle pose as to reduce the error associated with the control inputs. This assures the less error prone motion model producing faster convergence. We draw the superiority of fuzzy Kalman filtering for the state estimation through the SLAM algorithm developed with T-S fuzzy model in this paper. The proposed T-S fuzzy model based algorithm to the SLAM problem has proven that a demanding (not conventional) solution to the SLAM problem exists and it overcomes limitations of the EKF based SLAM, hinting a new path explored is much suitable in finding an advanced solution to localization and mapping problems.

2 Pseudolinear System Modeling

In the following, the vehicle state is defined by $\boldsymbol{x}_v = [x, y, \phi]^{\mathrm{T}}$, where x and y are the coordinates of the center of the rear axel of the vehicle with respect to some global coordinate frame and ϕ is the orientation of the vehicle axis. The landmarks are modeled as point landmarks and represented by a Cartesian pair such that $\boldsymbol{m}_i = [x_i, y_i]^{\mathrm{T}}, i = 1, ..., N$. Both vehicle and landmark states are registered in the same frame of reference.

2.1 The Pseudolinear Process Model

Figure 4 shows a schematic diagram of the vehicle in the process of observing a landmark. The pseudolinear vehicle process model in discrete time can be expressed as follows:

$$\boldsymbol{x}_v(k+1) = \boldsymbol{x}_v(k) + \boldsymbol{B}_v(k)\boldsymbol{u}_v(k) \quad (1)$$

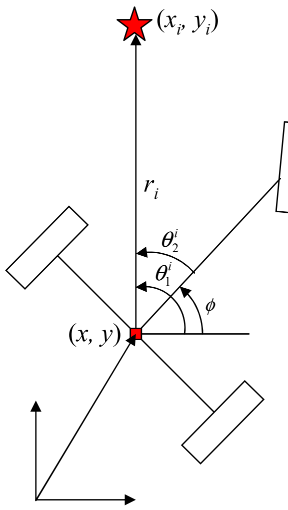


Figure 1: Vehicle in process of observing a landmark

for use in the prediction stage of the vehicle state estimator.

The landmarks in the environment are assumed to be stationary point targets. The landmark process model is thus

$$\begin{bmatrix} x_i(k+1) \\ y_i(k+1) \end{bmatrix} = \begin{bmatrix} x_i(k) \\ y_i(k) \end{bmatrix} \tag{2}$$

for all landmarks $i = 1, ..., N$. Equation (1) together with Eq. (2) defines the vehicle-landmarks process model.

2.2 The Observation Model with Two Sensors

Range $r_i(k)$ and two bearing measurements $\theta_1^i(k)$ and $\theta_2^i(k)$ to landmark i are recorded by the range and bearing sensors. The range measurements and bearing measurements are taken from the center of rear vehicle axel where the vehicle position (x, y) is taken. One sensor starts reading measurements from the x axis and the other from the center axis of the vehicle. Referring to Fig. 4, the observation model for ith landmark $\boldsymbol{z}_i(k) = [r_i(k), \theta_i(k), \beta_i(k)]^T$ can be written in a direct form as

$$r_i(k) = \sqrt{(x_i - x(k))^2 + (y_i - y(k))^2} + v_r(k) \tag{3}$$

$$\theta_i(k) = \theta_1^i(k) = \arctan\left(\frac{y_i - y(k)}{x_i - x(k)}\right) + v_{\theta_1}(k) \tag{4}$$

$$\theta_2^i(k) = \arctan\left(\frac{y_i - y(k)}{x_i - x(k)}\right) - \phi(k) + v_{\theta_2}(k) \tag{5}$$

$$\beta_i(k) = \theta_1^i(k) - \theta_2^i(k) = \phi(k) + v_{\theta_1}(k) - v_{\theta_2}(k) \tag{6}$$

where v_r and v_θ are the white noise sequences associated with the range and bearing measurements with zero means and standard deviations σ_r and σ_θ respectively. Equations (3), (5) and (6) define the observation model for the ith landmark.

2.3 Pseudolinear Observation Model

In this section, we present the pseudolinear measurement model. The pseudomeasurement method relies on representing the nonlinear measurement model (Eqs. (3), (5) and (6)) in the following pseudolinear form:

$$\boldsymbol{y}(\boldsymbol{z}) = \boldsymbol{H}(\boldsymbol{z})\boldsymbol{x} + \boldsymbol{v_y}(\boldsymbol{x}, \boldsymbol{v}) \tag{7}$$

Equations (3), (5) and (6) can be rearranged by algebraic and trigonometric manipulations to obtain the following model expressed by

$$r_i(k) = (x_i - x(k))\cos(\theta_i(k)) + (y_i - y(k))\sin(\theta_i(k)) + v_r(k)$$

$$0 = (x_i - x(k))\sin(\theta_i(k)) - (y_i - y(k))\cos(\theta_i(k)) + r_{i,\text{true}}(k)v_\theta(k)$$

$$\beta_i(k) = \phi(k) + v_{\theta_1}(k) - v_{\theta_2}(k) \tag{8}$$

The model (8) composed of above three equations can be expressed in the following pseudolinear form for the ith landmark:

$$\boldsymbol{y}(\boldsymbol{z}_i) = \begin{bmatrix} r_i(k) \\ 0 \\ \beta_i(k) \end{bmatrix} = \boldsymbol{H}(\boldsymbol{z}_i)\boldsymbol{x} + \boldsymbol{v_y}_i(\boldsymbol{x}, \boldsymbol{v}) \tag{9}$$

where the state vector is to be $\boldsymbol{x} = [\boldsymbol{x}_v^T \ \boldsymbol{m}_i^T \cdots \boldsymbol{m}_N^T]^T$ and $\boldsymbol{v_y}(\boldsymbol{x}, \boldsymbol{v})$ is considered to be white with its covariance expressed in the form:

$$\boldsymbol{R_y}(\hat{\boldsymbol{x}}) = \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \hat{r}_i^2\sigma_\theta^2 & 0 \\ 0 & 0 & \sigma_\beta^2 \end{bmatrix} \tag{10}$$

3 Formulation of Fuzzy Algorithm in SLAM Problem

To reduce the computational cost in using the T-S fuzzy model in SLAM problem, fuzzification of the process model and the pseudolinear measurement model is split into two cases according to the vehicle azimuth angle. A set of fuzzy rules is constructed for each case and is executed based on the initial separation of vehicle azimuth angle.

Case 1: If the azimuth angle of the vehicle ($\phi(t)$) lies between $-\pi/2$ and $\pi/2$, the jth rule for this case will be of the form:

Local linear system rule j:
IF $\phi(t)$ is F_ϕ^j and $\theta_i(t)$ is F_θ^j THEN

$$\boldsymbol{x}_j(k+1) = \boldsymbol{x}(k) + \boldsymbol{B}_j(k)\boldsymbol{u}(k) \quad \text{for} \quad j = 1, 2, \cdots, 8$$

$$\boldsymbol{y}_{ij}(k+1) = \boldsymbol{H}_{ij}(k+1)\boldsymbol{x}_j(k+1) + \boldsymbol{v}^i_{\boldsymbol{y}_j}(k+1) \quad (11)$$

F^j_ϕ, F^j_θ are the fuzzy sets of vehicle azimuth angel and bearing angle for the jth rule respectively.
Case 2: It is defined for $\pi/2 < |\phi(t)| < \pi$ and will be composed of eight similar local linear models as defined above. The fuzzy Kalman filter (FKF) algorithm proceeds recursively in the three stages:

- Prediction:
  The algorithm first generates a prediction for the state estimate, the observation (relative to the ith landmark) and the state estimate covariance at the time $k+1$ for the jth rule according to

$$\hat{\boldsymbol{x}}_j(k+1|k) = \hat{\boldsymbol{x}}(k|k) + \boldsymbol{B}_j(k)\boldsymbol{u}(k) \quad (12)$$

$$\hat{\boldsymbol{y}}_{ij}(k+1|k) = \boldsymbol{H}_{ij}(k+1)\hat{\boldsymbol{x}}_j(k+1|k) \quad (13)$$

$$\boldsymbol{P}_j(k+1|k) = \boldsymbol{P}(k|k) + \boldsymbol{B}_j(k)\boldsymbol{Q}(k)\boldsymbol{B}^T_j(k) \quad (14)$$

- Observation:
  Following the prediction, the observation $\boldsymbol{y}_i(k+1)$ of the ith landmark of the true state $\boldsymbol{x}(k+1)$ is made according to Eq. (9). Assuming correct landmark association, an innovation is calculated for the jth rule as follows:

$$\boldsymbol{v}_{ij}(k+1) = \boldsymbol{y}_{ij}(k+1) - \hat{\boldsymbol{y}}_{ij}(k+1|k) \quad (15)$$

  together with an associated innovation covariance matrix for the jth rule given by

$$\boldsymbol{S}_{ij}(k+1) = \boldsymbol{H}_{ij}(k+1)\boldsymbol{P}_j(k+1|k)\boldsymbol{H}^T_{ij}(k+1) + \boldsymbol{R}_{ij}(k+1) \quad (16)$$

- Update:
  The state update and corresponding state estimate covariance are then updated for the jth rule according to

$$\hat{\boldsymbol{x}}_j(k+1|k+1) = \hat{\boldsymbol{x}}_j(k+1|k) + \boldsymbol{K}_j(k+1)\boldsymbol{v}_{ij}(k+1) \quad (17)$$

$$\boldsymbol{P}_j(k+1|k+1) = \boldsymbol{P}_j(k+1|k) - \boldsymbol{K}_j(k+1)\boldsymbol{S}_{ij}(k+1) \times \boldsymbol{K}^T_j(k+1) \quad (18)$$

  Here the gain matrix $\boldsymbol{K}_j(k+1)$ is given by

$$\boldsymbol{K}_j(k+1) = \boldsymbol{P}_j(k+1|k)\boldsymbol{H}^T_{ij}(k+1)\boldsymbol{S}^{-1}_{ij}(k+1) \quad (19)$$

Local state estimates are then combined to obtain the global state estimate for the T-S fuzzy model given by Eq. (11). The global estimate is then obtained by the following equation:

$$\hat{\boldsymbol{x}}(k+1|k+1) = \sum_{j=1}^{8} h_j(z_i(k))\hat{\boldsymbol{x}}_j(k+1|k+1) \quad (20)$$

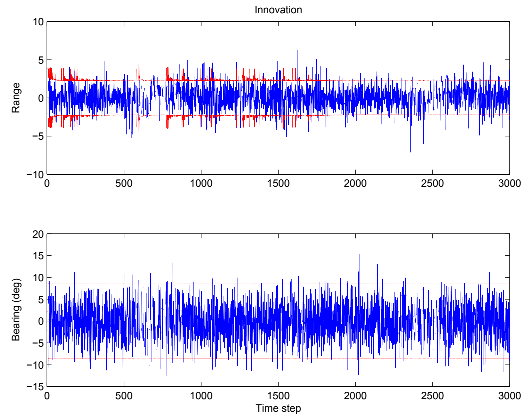


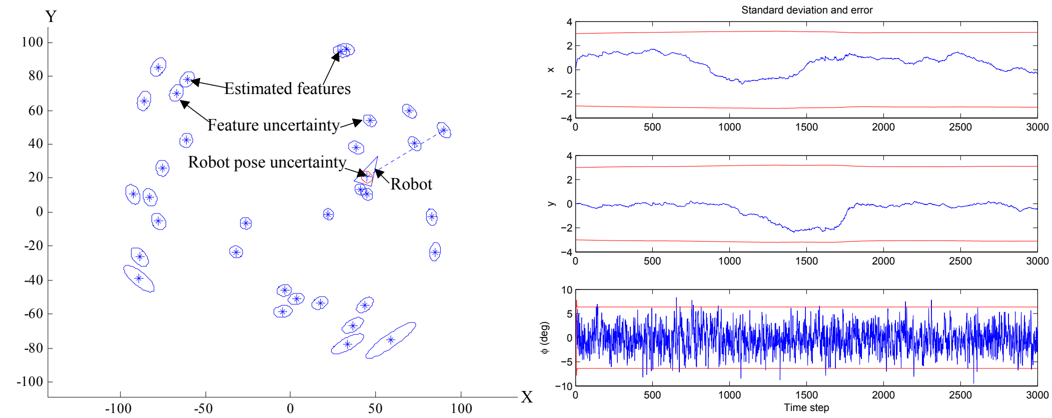
Figure 2: Innovation in measurements

where $z_i(k) = [z_{i1}(k) \;\; z_{i2}(k)] = [\phi(k) \;\; \theta_i(k)]$. The common covariance can be formulated as follows:

$$\boldsymbol{P}(k+1|k+1) = \min(\text{trace}\,(\boldsymbol{P}_j(k+1|k+1)))\;\forall j \quad (21)$$

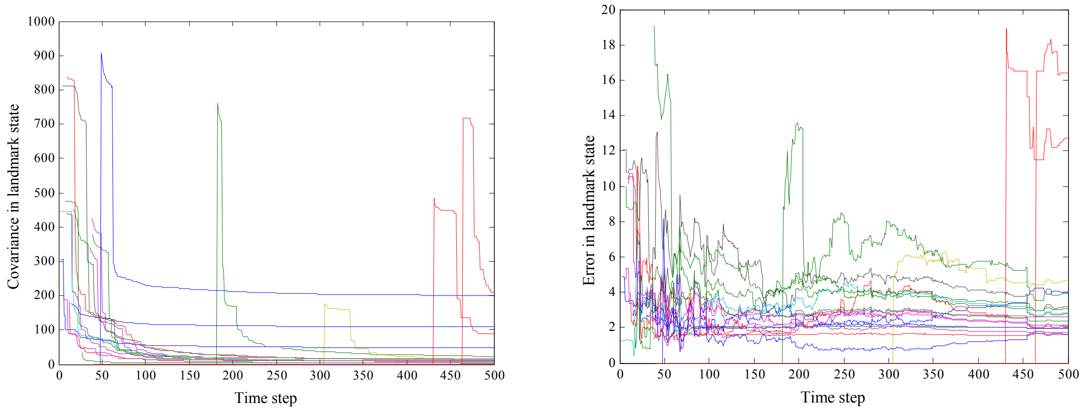
4 Simulation Results

The newly proposed method is applied to the feature based SLAM. The developed algorithm was simulated for the system composed of Eqs. (1), (2) and (9). An environment populated with point landmarks was simulated with the FKF algorithm to generate the state estimates and state errors. Simulation results are depicted in Fig. 2, Fig. 3 and Fig. 4. Innovations are the only available measure to examine online filter behavior when true state values are unavailable. Innovations here (see Fig. 2) indicate that the proposed filter and the models are consistent. Figure 3 shows localization of the vehicle and map building simultaneously over time. Figure 3(a) shows the estimated map built over time. It can be seen that error ellipses of the features are getting converged to actual landmark locations. It is clear that the newly proposed algorithm can well map the environment. Figure 3(b) shows standard deviation and error associated with the vehicle pose. It can be seen that the vehicle localization is performed well by the newly presented method as vehicle pose error is decreasing to a minimum bound gradually. Figure 4(a) shows the evolution of the landmark uncertainty and it can be observed that the landmark uncertainty is gradually decreasing over time. Figure 4(b) shows the evolution of landmark state error and it is once proved that the proposed method works well in SLAM problem. It is observed that the landmark state error obtained from the pseudolinear model based FKF approach reaches to a minimum bound within a shorter time



(a) (b)

Figure 3: Simultaneous localization and map building



(a) (b)

Figure 4: Evolution of landmark state covariance and error

steps compared to that obtained from the EKF algorithm. Note that the results obtained from the EKF-based SLAM are not shown in this paper due to space limitations.

5 Conclusion

A fuzzy logic and pseudolinear model based solution to the SLAM problem was first proposed in this paper and validity of the method was proved with simulation results. The need for direct linearization of nonlinear systems for state estimation is diminished as the newly proposed method performed well and provided a better solution to the SLAM problem. Results obtained from the newly introduced method were compared with the results obtained from widely used EKF algorithm to highlight the merit of the pseudolinear model based system with fuzzy logic. It was proved that the pseudolinear model based fuzzy Kalman filter algorithm provided more satisfactory results over the EKF because the pseudolinear models did not lose its nonlinearity when employed in the Kalman filter equations. It was found that a fuzzy logic based approach with the pseudolinear models provided a remarkable solution to state estimation process because fuzzy logic has been always standing for a better solution.

References

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